

Effect of Size on Ground-Coupled Heat Pump Performance

by

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Thesis submitted in partial fulfillment of  
the requirements for the degree of  
Master of Science in the Department of  
Mechanical Engineering and Materials Science in the Graduate School  
of Duke University

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ABSTRACT

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## **Abstract**

Here we document and explain a general trend in the performance of refrigeration and heat pump systems: larger installations are more efficient. We show analytically why the performance of the system must increase with the size of the installation. The second law efficiency of refrigeration systems must increase with their size. We also show that the power requirement a ground-coupled heat pump system must decrease as the size of the ground heat exchanger increases. From these two trends emerges the trade off between the size of the heat pump and the size of the ground heat exchanger. The challenge is to find the optimum size of the ground-coupled heat pump. We show numerically the optimum heat pump size and the ground heat exchanger size that correspond to minimum total power requirement subject to a cost constraint.

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## Nomenclature

$A$	heat transfer surface area [ $\text{m}^2$ ]
$\tilde{A}$	dimensionless size parameter
$B$	dimensionless parameter
$C$	empirical constant [W]
$C_i$	internal heat leak constant [W/K]
$\text{COP}$	coefficient of performance
$\text{COP}_{\text{rev}}$	coefficient of performance in the reversible limit
$\text{COP}_{\text{HP,rev}}$	heat pump coefficient of performance in the reversible limit
$K$	total cost constraint [\$]
$\tilde{K}$	dimensionless total cost constraint
$P$	probability
$p_A$	price of the underground heat exchanger [\$]
$p_q$	price of the load [\$]
$q_0$	heat rejected to ambient [W]
$q_{0c}$	heat rejected from compartment without irreversibility [W]
$q_H$	heat pumps to the building [W]
$q_i$	internal heat leak [W]
$q_L$	refrigerator load [W]
$\tilde{q}_L$	dimensionless size parameter



$q_{LC}$	refrigerator compartment without irreversibility [W]
$R$	coefficient of determination
$S_{gen,H}$	entropy generation through building insulation [W/K]
$S_{gen,HP}$	entropy generated by the heat pump [W/K]
$S_{gen,L}$	entropy generation through the ground heat exchanger [W/K]
$T_0$	ambient temperature [K]
$T_H$	hot-end temperature [K]
$T_L$	cold-end temperature [K]
$U$	overall heat transfer coefficient [W/m <sup>2</sup> .K]
$W$	consumed work [W]
Greek symbols	
$\varepsilon$	dimensionless parameter
$\eta_{II}$	second law efficiency
$\pi$	dimensionless price ratio

# 1. Introduction

Refrigeration, heating and air conditioning are technologies that have played a key role in increasing economic activity during the past century [1]. Regions of the globe historically sleepy because of extreme warmth or cold were brought to “temperate” climate, life style and productivity by this technology. Today, this technology is so prevalent and so useful that, like power generation, it is taken for granted.

In this paper we take a fresh and fundamental look at how to best distribute the beneficial effect of this technology while both maximizing efficiency and minimizing cost. We start by looking at the area that is served by the technology, and the size of the heat pump installation that serves this area. We show that the size effect is paramount. Larger systems, and larger flow systems in general are more efficient thermodynamically than smaller systems [2-17]. Ignoring cost considerations, the most efficient design is a landscape served by a few large installations, each installation allocated to an inhabited space (area). The cost of ground-coupled heat pump systems depends primarily on the both the size of the heat pump and the size of the ground heat exchanger [18]. In this paper we develop a method to determine the optimum size of the heat pump that results in minimum total power consumption under a cost constraint.

## 2. Model

Consider the refrigerator model shown in Fig. 1a. The high temperature ( $T_0$ ) represents the ambient, and the low temperature ( $T_L$ ) belongs to the cold space, for example, the interior of a building for which the refrigerator provides air conditioning. The refrigerator is a closed system operating in steady state or in an integral number of cycles, and this means that it generates entropy at a constant rate,

$$S_{\text{gen}} = \frac{q_0}{T_0} - \frac{q_L}{T_L} > 0 \quad (1)$$

Alternatively, it means that the coefficient of performance

$$\text{COP} = \frac{q_L}{W} \quad (2)$$

is smaller than in the limit of reversible operation,

$$\text{COP}_{\text{rev}} = \frac{q_L}{W_{\text{rev}}} = \frac{T_L}{T_0 - T_L} \quad (3)$$

The second-law efficiency of the refrigeration system is the ratio of the actual coefficient of performance and the reversible coefficient of performance,

$$\eta_{\text{II}} = \frac{\text{COP}}{\text{COP}_{\text{rev}}} < 1 \quad (4)$$

This ratio is less than 1, as shown by the data compiled in Fig. 2 [2]. The  $\eta_{\text{II}}$  data also show that the size of the refrigeration installation ( $q_L$ ) has a significant effect on thermodynamic performance: larger installations are more efficient. In this section we show why  $\eta_{\text{II}}$  should be expected to increase with the size of the installation.

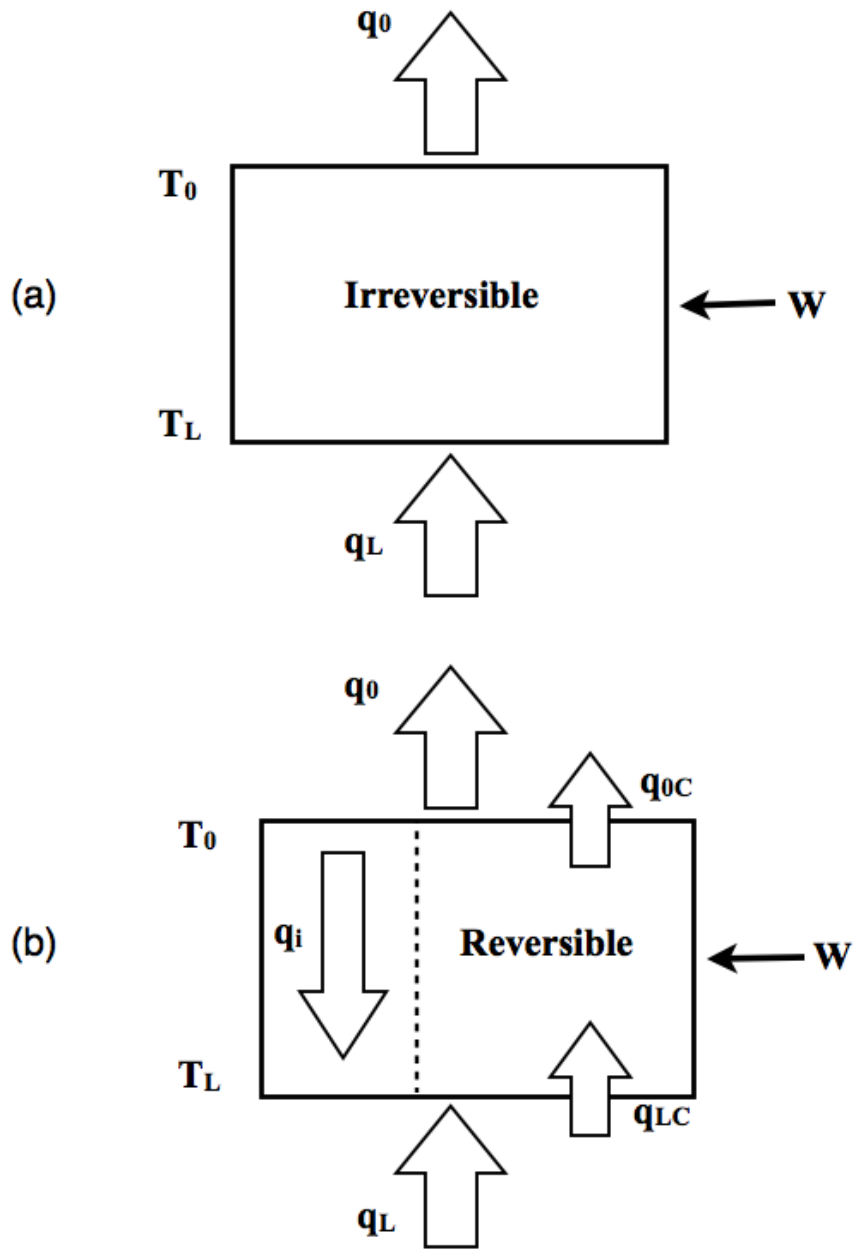


Figure 1: (a) Irreversible refrigerator, and (b) Refrigerator model with internal heat leak irreversibility.

In Fig. 1b we propose a two-part model that accounts for the irreversible operation of the refrigerator system. The refrigerator irreversibility is due to the heat current  $q_i$  that leaks through the system from the ambient  $T_0$  to the cold space  $T_L$ . The internal heat leak is modeled as

$$q_i = C_i(T_0 - T_L) \quad (5)$$

where  $C_i$  represents the internal heat conductance of the refrigerator. The rest of the refrigerator is modeled as reversible (Carnot), and it operates in parallel with the internal heat leak. For the reversible part, the second law of thermodynamics requires

$$\frac{q_{LC}}{T_L} = \frac{q_{0C}}{T_0} \quad (6)$$

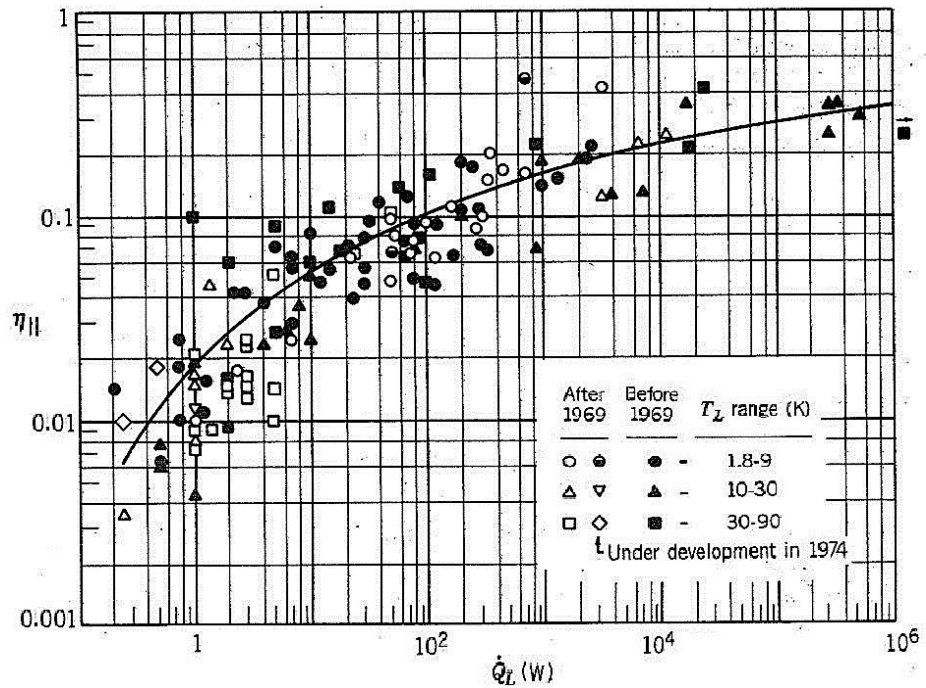


Figure 2: Second-law efficiencies of refrigerators and liquefiers [2].

The refrigeration load ( $q_L$ ) removed from the cold space is

$$q_L = q_{LC} - q_i \quad (7)$$

The coefficient of performance of Eq. (2) can be expressed in terms of the actual size of the refrigerator ( $q_L$ ) as follows:

$$\begin{aligned} \text{COP} &= \frac{q_L}{q_0 - q_L} = \frac{q_L}{q_{0C} - q_{LC}} \\ &= \frac{q_{LC}}{q_{0C} - q_{LC}} \cdot \frac{q_L}{q_{LC}} = \frac{T_L}{T_0 - T_L} \cdot \frac{q_L}{q_{LC}} \\ &= \text{COP}_{\text{rev}} \frac{q_L}{q_{LC}} \end{aligned} \quad (8)$$

The second law efficiency [Eq. (4)] becomes

$$\eta_{II} = \frac{\text{COP}}{\text{COP}_{\text{rev}}} = \frac{q_L}{q_L + q_i} \quad (9)$$

and after using Eq. (5),

$$\eta_{II} = \frac{1}{1 + C_i \frac{T_0}{q_L} \left(1 - \frac{T_L}{T_0}\right)} \quad (10)$$

The challenge is to account for the effect that the refrigeration size  $q_L$  has on the second law efficiency. First, we rewrite Eq. (10) as

$$C_i T_0 = \frac{\left(\frac{1}{\eta_{II}} - 1\right) q_L}{1 - \frac{T_L}{T_0}} \quad (11)$$

Next, we substitute on the right hand side of Eq. (11) the data ( $\eta_{II}$ ,  $q_L$ ,  $T_L$ ) read off Fig. 2.

For the  $T_L$  value in Eq. (11) we use the average of the  $T_L$  range indicated in Fig. 2. The

$C_i T_0$  value calculated with Eq. (11) is plotted with respect to refrigeration size ( $q_L$ ) as shown in Fig. 3. The trend revealed by Fig. 3 is correlated by

$$C_i T_0 = 61 q_L^{0.68}, \quad (R^2 = 0.932, P = 0.086) \quad (12)$$

This correlation is statistically significant because the P value is less than 0.1 [19, 20]. In conclusion, the second law efficiency correlated for the performance data of Fig. 2 is

$$\eta_{II} = \left[ 1 + \left( \frac{q_L}{C} \right)^{-0.32} \left( 1 - \frac{T_L}{T_0} \right) \right]^{-1} \quad (13)$$

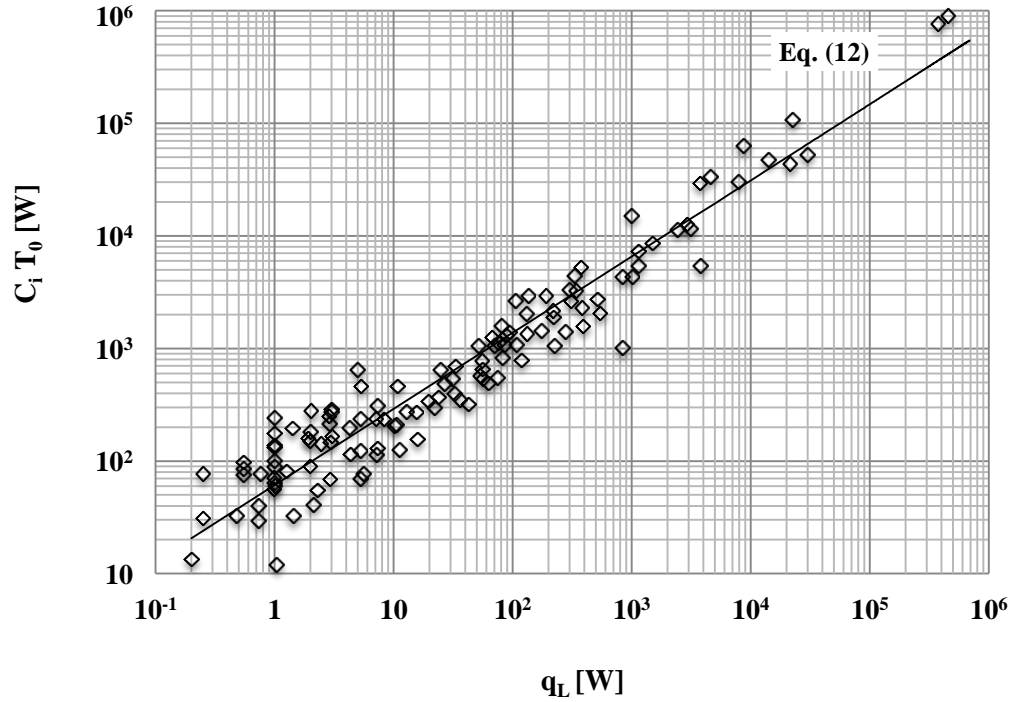


Figure 3: The effect of refrigerator size on performance.

where  $C$  is an empirical constant equal to  $3.8 \times 10^6$  W. This analytical form shows that larger installations are more efficient. This is in accord with the effect of size on the performance of power plants [1, 4], which is reinforced by Fig. 4 where the  $\eta_{II}$  value from Fig. 2 is compared favorably with the  $\eta_{II}$  value calculated with Eq. (13)

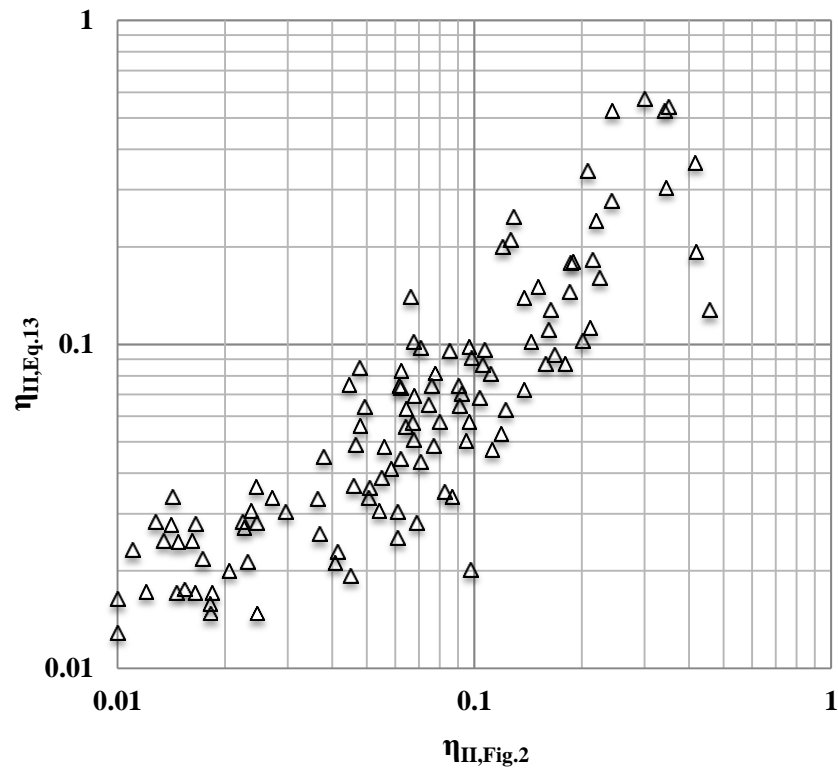


Figure 4: Comparison between the  $\eta_{II}$  data of Fig. 2 and the  $\eta_{II}$  values calculated with Eq. (13).



### 3. Economies of scale

The fact that larger installations are more efficient means that as individual refrigeration needs increase over time we witness a trend toward central installations that supply cooling to many users. The advantage of switching from individual refrigeration to group refrigeration is made visible by analyzing the two designs shown in Fig. 5. In both designs the refrigeration load is  $2q_L$ . In the individual design there are two refrigerators, each requiring the power input

$$W_1 = \frac{q_L}{\text{COP}} = \frac{q_L}{\text{COP}_{\text{rev}}} \left[ 1 + \left( \frac{q_L}{C} \right)^{-0.32} \left( 1 - \frac{T_L}{T_0} \right) \right] \quad (14)$$

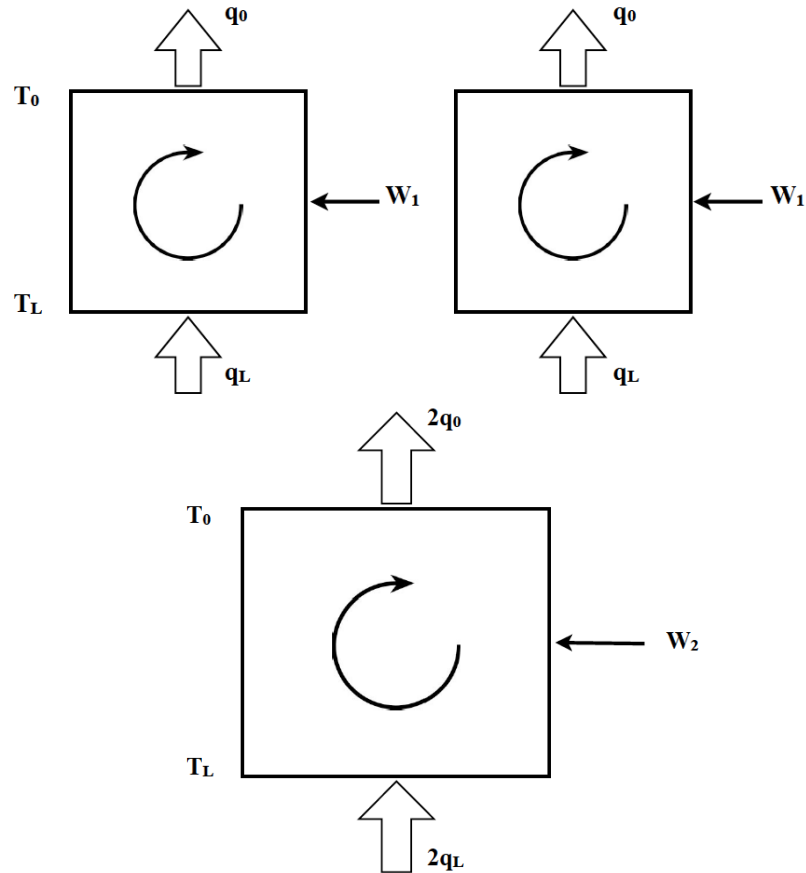


Figure 5: Individual refrigeration (top) vs. group refrigeration (bottom).

In the group refrigeration scheme there is a single installation requiring

$$W_2 = \frac{2 q_L}{COP_{rev}} \left[ 1 + \left( \frac{2q_L}{C} \right)^{-0.32} \left( 1 - \frac{T_L}{T_0} \right) \right] \quad (15)$$

The comparison between two individual designs and one group design is the ratio

$$\frac{2 W_1}{W_2} = \frac{1 + \left( \frac{q_L}{C} \right)^{-0.32} \left( 1 - \frac{T_L}{T_0} \right)}{1 + \left( \frac{2q_L}{C} \right)^{-0.32} \left( 1 - \frac{T_L}{T_0} \right)} \quad (16)$$

This ratio is always greater than 1. For example, when  $q_L$  is small enough this ratio reduces to

$$\frac{2 W_1}{W_2} = \frac{q_L^{-0.32}}{(2 q_L)^{-0.32}} = 1.25 \quad (17)$$

In this limit, the savings brought by the group design represent a savings of 20% relative to using the individual designs.

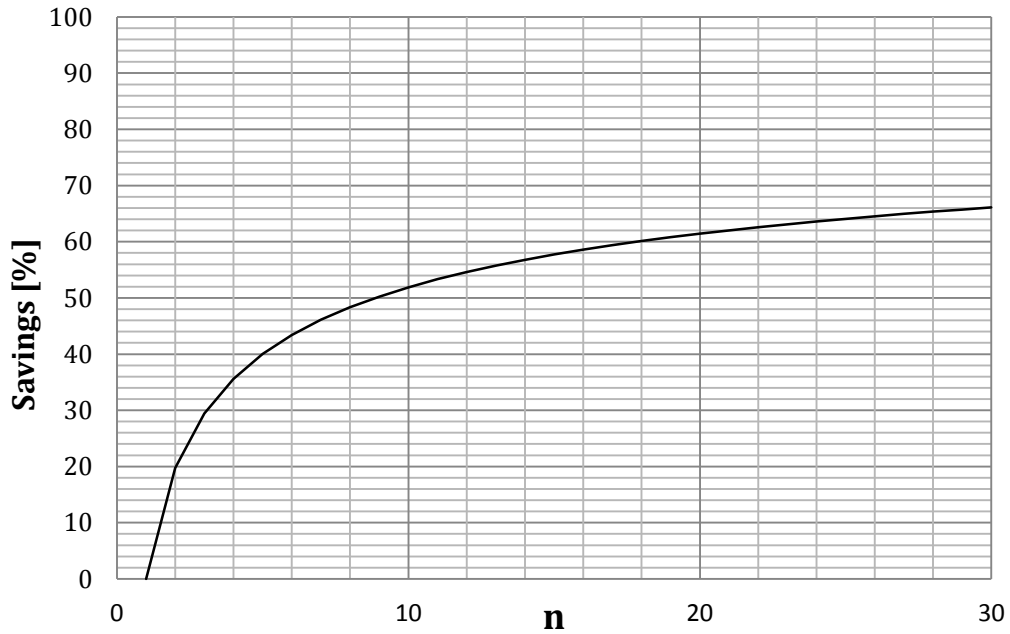
The preceding analysis can be generalized to  $n$  number of users. The power input required by each individual design is the same as in Eq. (14). In the group refrigeration, there is a single installation requiring

$$W_n = \frac{n q_L}{COP_{rev}} \left[ 1 + \left( \frac{n q_L}{C} \right)^{-0.32} \left( 1 - \frac{T_L}{T_0} \right) \right] \quad (18)$$

The percentage of savings is indicated by

$$\begin{aligned} \% \text{ Savings} &= \left[ 1 - \frac{W_n}{n W_1} \right] \times 100 \\ &= \left[ 1 - \frac{1 + \left( \frac{n q_L}{C} \right)^{-0.32} \left( 1 - \frac{T_L}{T_0} \right)}{1 + \left( \frac{q_L}{C} \right)^{-0.32} \left( 1 - \frac{T_L}{T_0} \right)} \right] \times 100 \end{aligned} \quad (19)$$

Figure 6 shows the percentage of savings recorded after switching from individual units to central refrigeration for  $n$  users. For small enough  $q_L$ , the percentage of savings reaches the limit represented by the trend line. For installations that are large enough (i.e. with large  $q_L$ ), the percentage of savings approaches zero, because the ratio  $W_n/nW_1$  approaches 1. The savings increase as  $n$  increases, but the rate of increase becomes smaller as  $n$  increases. In conclusion, the percentage of savings depends on the size of the installation.



**Figure 6: The percentage of savings limit by switching from individual refrigeration to group refrigeration for small enough  $q_L$ , Eq. (17).**

## 4. Heat pump in winter

When the refrigeration plant is used in an application that requires cooling or heating, the thermodynamic losses are more numerous and greater than the losses modeled in the proceeding section and Fig. 1. Consider the example of a ground-coupled heat pump system that heats a building ( $T_H$ ) in winter ( $T_0$ ). As shown in Fig. 7, the building loses heat at the rate  $q_H$  to the ambient, and the heat pump delivers  $q_H$  to the building. At the cold end the heat pump extracts  $q_L$  from the ground heat exchanger, while the same  $q_L$  leaks from the volume of soil into the heat exchanger. There are three sources of entropy generation in this installation, the heat pump itself,

$$S_{\text{gen,HP}} = \frac{q_H}{T_H} - \frac{q_L}{T_L} \quad (20)$$

the heat leakage through the building insulation

$$S_{\text{gen,H}} = \frac{q_H}{T_0} - \frac{q_H}{T_H} \quad (21)$$

and the heat transfer from the ground to the cold water that circulates through the heat exchanger (an assembly of pipes) buried in the ground,

$$S_{\text{gen,L}} = \frac{q_L}{T_L} - \frac{q_L}{T_0} \quad (22)$$

After absorbing  $q_L$  from the ground heat exchanger, the circulating water delivers  $q_L$  to the working fluid that circulates inside the heat pump.

All three contributions to the entropy generation rate depend on the physical size of their respective components in the greater system. For example, the size effect on the entropy generation rate of the heat pump was documented in section 2. Another

example is the ground heat exchanger, where the heat transfer rate ( $q_L$ ) is specified by the need to heat the building,

$$q_L = U A (T_0 - T_L) \quad (23)$$

where  $A$  is the heat transfer surface of the heat exchanger, and  $U$  is the overall heat transfer coefficient that accounts for periodic thermal diffusion in the ground that makes contact with  $A$ . Eliminating  $T_L$  between Eqs. (22) and (23) we find that  $S_{\text{gen},L}$  decreases as the size  $A$  increases,

$$S_{\text{gen},L} = \frac{q_L}{T_0} \left( \frac{U A T_0}{q_L} - 1 \right)^{-1} \quad (24)$$

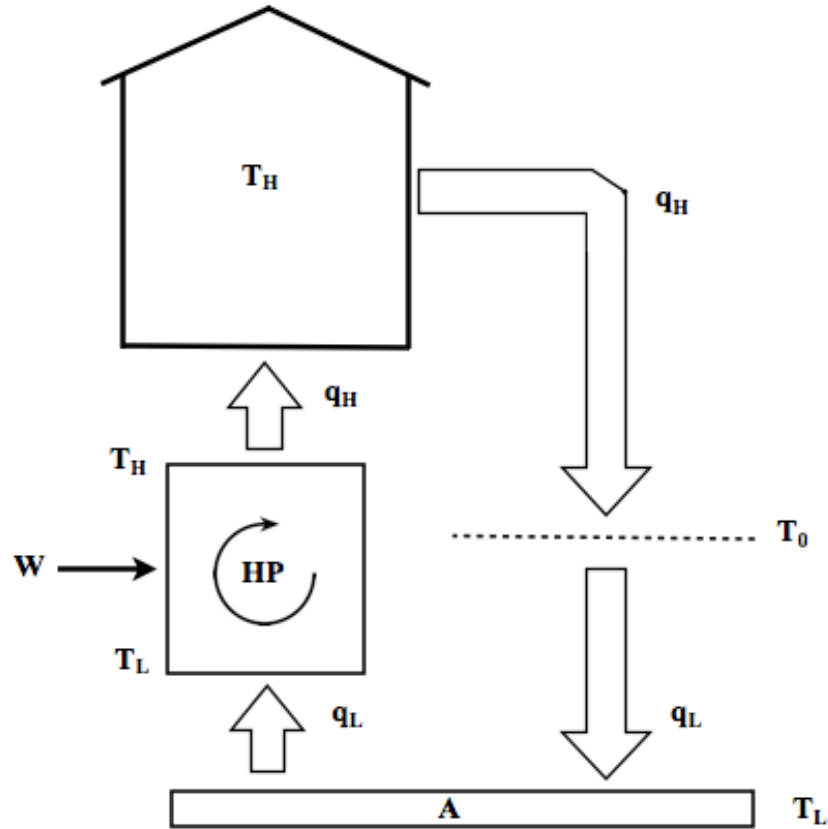


Figure 7: Heat pump in winter.

In view of the  $\eta_{II}$  definition, Eq. (4), the entropy generated by the heat pump alone becomes

$$S_{\text{gen,HP}} = \frac{q_L}{T_L} \left( 1 - \frac{T_L}{T_H} \right) \left( \frac{1}{\eta_{II}} - 1 \right) \quad (25)$$

The behavior of  $S_{\text{gen,HP}}$  is the same as that of  $S_{\text{gen,L}}$ : both increase in proportion with the load ( $q_L$ ), however the size effect (larger  $\eta_{II}$ , or larger  $A$ ) slows down this increase.

In the overall scheme (Fig. 7), the power requirement  $W$  depends on the size of the heat pump (indicated by  $q_L$ ) and the size of the heat transfer area  $A$ , under the constraint that  $q_H$  is fixed. By combining Eqs. (2), (4) and (23), while noting that for the heat pump  $\text{COP}_{\text{HP,rev}} = T_L/(T_H - T_L)$ , the inverse of the power requirement becomes

$$\frac{q_H}{W} = 1 + \eta_{II} \frac{1 - B}{\varepsilon + B} \quad (26)$$

where

$$\varepsilon = \frac{T_H - T_0}{T_0} , \quad B = \frac{q_L}{UAT_0} \quad (27)$$

For  $\eta_{II}$  we use the correlation (13) with  $T_H$  in place of  $T_0$ ,

$$\eta_{II} = \left[ 1 + \left( \frac{q_L}{C} \right)^{-0.32} \left( 1 - \frac{T_L}{T_H} \right) \right]^{-1} \quad (28)$$

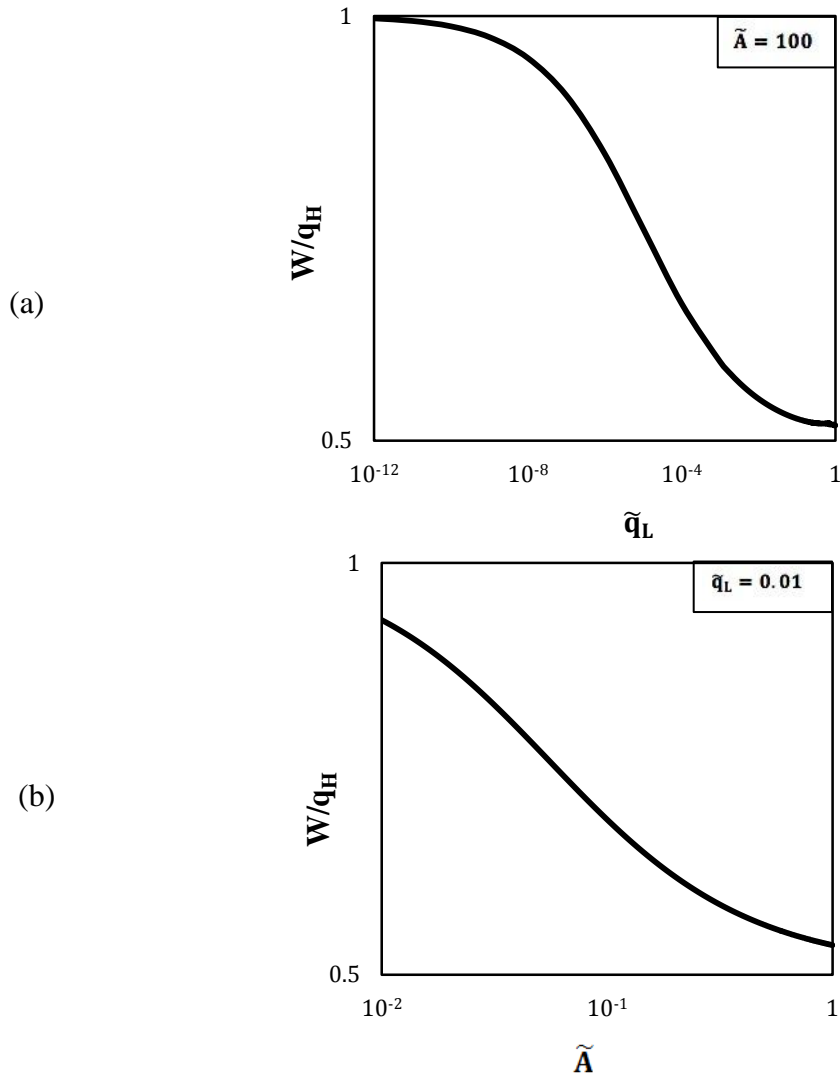
and obtain

$$\eta_{II} = \left[ 1 + \left( \frac{q_L}{C} \right)^{-0.32} (\varepsilon + B) \frac{T_0}{T_H} \right]^{-1} \quad (29)$$

Equations (26) and (29) show that the ratio  $W/q_H$  depends on two dimensionless “size” parameters

$$\tilde{q}_L = \frac{q_L}{C} \quad \text{and} \quad \tilde{A} = \frac{UAT_0}{C} \quad (30)$$

where  $B = \tilde{q}_L/\tilde{A}$ , and on one constant,  $\varepsilon$  or  $T_0/T_H$ , cf. Eq. (27). Figure 8 shows that the ratio  $W/q_H$  decreases as either  $\tilde{q}_L$  or  $\tilde{A}$  increases.



**Figure 8: The variation of the total power requirement  $W/q_H$  with the heat pump size  $\tilde{q}_L$ , and the ground heat exchanger size  $\tilde{A}$ .**

## 5. Economies of scale: Heat pump in winter

Consider the competition between two heat pump designs, each providing the heat input  $2q_H$  to an inhabited space that leaks  $2q_H$  to the ambient. As shown in Fig. 9, one design consists of two identical heat pumps each delivering  $q_H$  to the inhabited space. According to Eqs. (26) and (29), the power required by one heat pump ( $W_1$ ) is given by

$$\frac{q_H}{W_1} = 1 + \frac{1 - B}{\varepsilon + B} \left[ 1 + \left( \frac{q_L}{C} \right)^{-0.32} (\varepsilon + B) \frac{T_0}{T_H} \right]^{-1} \quad (31)$$

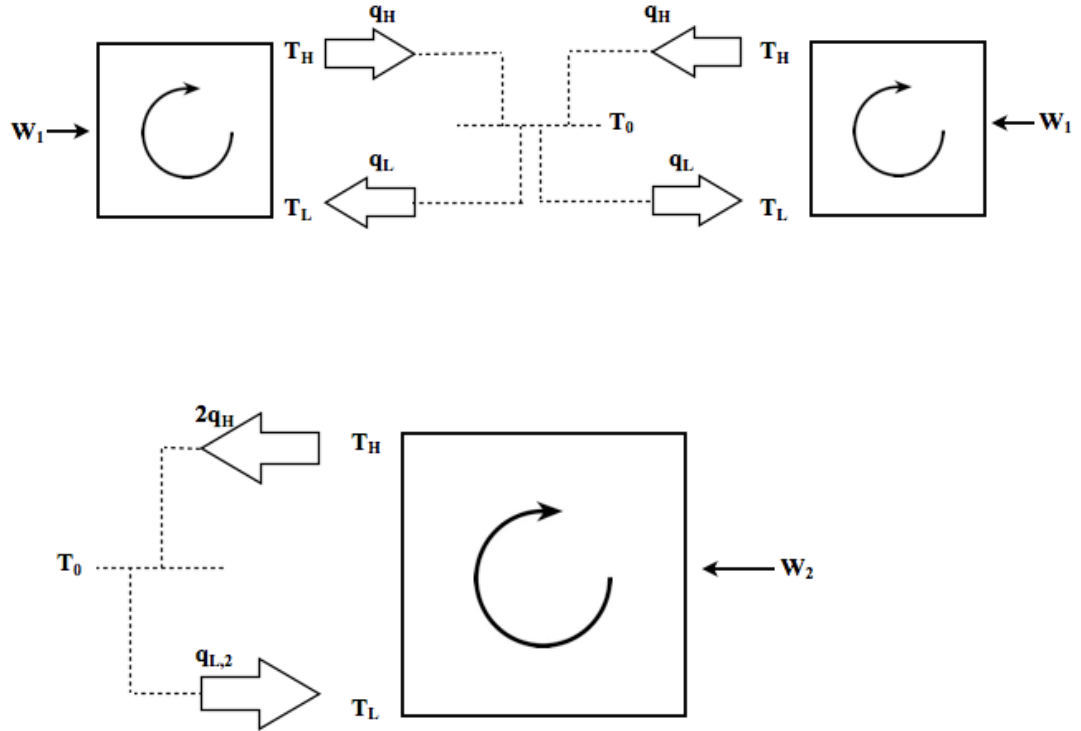


Figure 9: Individual heat pump (top) vs. group heat pump (bottom).



Likewise, the power ( $W_2$ ) required by the single heat pump that delivers  $2q_H$  is given by

$$\frac{2 q_H}{W_2} = 1 + \frac{1 - B}{\varepsilon + B} \left[ 1 + \left( \frac{q_{L,2}}{C} \right)^{-0.32} (\varepsilon + B) \frac{T_0}{T_H} \right]^{-1} \quad (32)$$

where  $q_{L,2}$  is comparable with  $2q_L$ . The relative merit of the two designs is measured as the ratio  $2W_1/W_2$ , which is obtained by dividing Eq. (32) by Eq. (31). In the limit  $q_L/C \gg 1$ , the ratio  $2W_1/W_2$  approaches 1. In the opposite limit  $q_L/C \ll 1$ , the ratio  $2W_1/W_2$  again approaches 1. In between, for example when  $q_L/C = 1$ , and after assuming  $T_L/T_0 = 0.95$  and  $1 - B \cong 1$ , the ratio  $2W_1/W_2$  is equal to 1.12, which means that the saving represented by the group design are of the order of 12% relative to using the individual designs. The conclusion is that the centralized heating scheme is more efficient than the individual heating scheme, and the greatest benefit of economies of scale is felt when  $q_L/C$  is of order 1.

The advantage of using central heat pump for the entire building or multiple heat pumps can be made visible by analyzing the three designs shown in Fig. 10. Consider a ground-coupled heat pump that provides heating to a building complex that consist of four identical units, as shown in Fig. 10a. The heat current from the ground to the piping buried in the ground passes through the heat transfer surface area  $A$ . The heat pump is required to deliver  $q_H$  to the building, and this heat current is distributed to the four identical units. The cost of a ground coupled heat pump system depends on many factors, including the heating or cooling load and the size of the heat exchanger surface. The power required by the heat pump is given by

$$\frac{q_H}{W_1} = 1 + \frac{1-B}{\varepsilon+B} \left[ 1 + \left( \frac{q_L}{C} \right)^{-0.32} (\varepsilon+B) \frac{T_0}{T_H} \right]^{-1} \quad (33)$$

Since the building is divided into four zones, multiple heat pumps can also be used. In Fig. 10b we consider heat pumps, each providing heating ( $q_H/2$ ) to two units, and each with a heat exchanger surface area  $A/2$ . The power required by each heat pump will be

$$\frac{q_H/2}{W_{1/2}} = 1 + \frac{1-B}{\varepsilon+B} \left[ 1 + \left( \frac{q_L}{2C} \right)^{-0.32} (\varepsilon+B) \frac{T_0}{T_H} \right]^{-1} \quad (34)$$

A third alternative is to heat the building with four heat pumps, one heat pump for each unit. The heat input provided by each heat pump is  $q_H/4$  by using underground heat transfer area  $A/4$ . The power consumed by each will be

$$\frac{q_H/4}{W_{1/4}} = 1 + \frac{1-B}{\varepsilon+B} \left[ 1 + \left( \frac{q_L}{4C} \right)^{-0.32} (\varepsilon+B) \frac{T_0}{T_H} \right]^{-1} \quad (35)$$

The three designs of Fig. 10 compete as follows. If  $q_L/C \cong 1$  and  $T_L/T_0 \cong 0.95$ , then the relative performance of Figs. 10a and 10b is represented by

$$\frac{2W_{1/2}}{W_1} = 1.12 \quad (36)$$

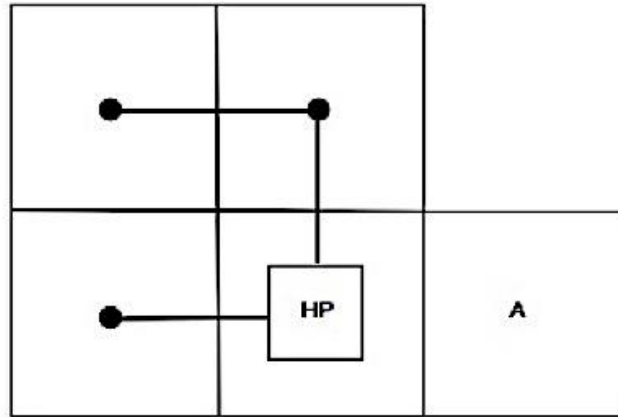
The savings associated with the central heat pump design are 12% relative to using two heat pumps.

Similarly, the power requirement of four individual heat pumps (Fig. 10c) relative to one central heat pump (Fig. 10a) in the case of  $q_L/C \cong 1$  and  $T_L/T_0 \cong 0.95$  is

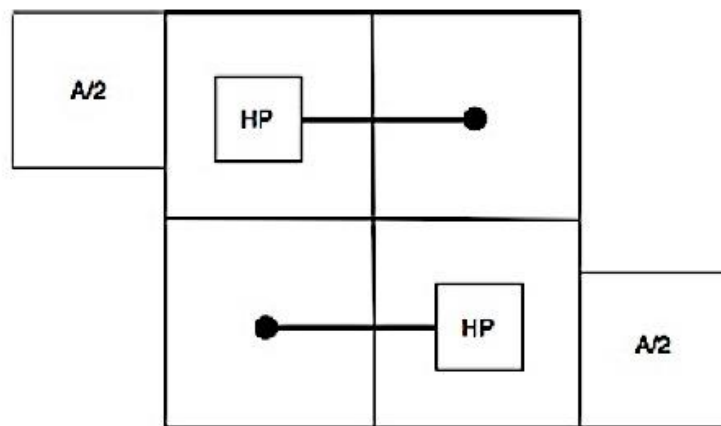
$$\frac{4W_{1/4}}{W_1} = 1.22 \quad (37)$$

The advantage of switching from four individual heat pumps to a central heat pump design yields 22% in savings. Generally, a central heat pump is more advantages than using multiple heat pumps.

(a)



(b)



(c)

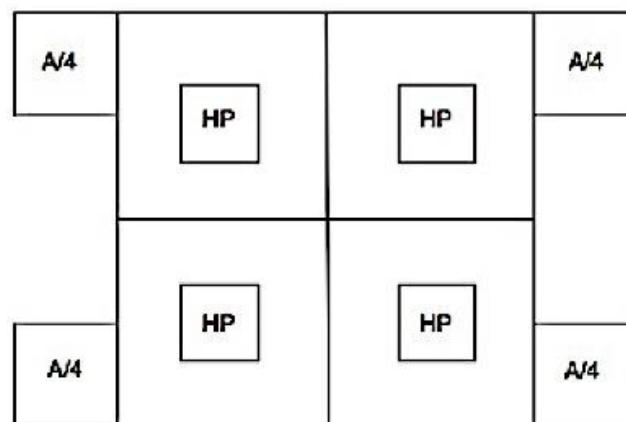


Figure 10: Central heat pump design vs. multiple heat pumps.

## 6. Discussion

In this paper we showed that the total power requirement of a ground coupled heat pump system depends on size, Eqs. (26) – (30):  $W/q_H$  decreases as the size of the heat pump ( $\tilde{q}_L$ ) or the size of the ground heat exchanger ( $\tilde{A}$ ) increases, as shown in Fig. 8. These trends compete for a lower overall  $W/q_H$ , and lead to the question of which “size” has more impact on decreasing the power requirement,  $\tilde{q}_L$  or  $\tilde{A}$ . The related design question is how to size the heat pump ( $\tilde{q}_L$ ) relative to its ground heat exchanger ( $\tilde{A}$ ).

The allocation of size is based on combining the thermodynamics of section 4 with economics. Both sizes,  $\tilde{q}_L$  and  $\tilde{A}$ , participate in an additive total cost constraint of the type

$$K = p_q \tilde{q}_L + p_A \tilde{A} \quad (38)$$

where the prices  $p_q$  and  $p_A$  represent the costs per unit of  $\tilde{q}_L$  and  $\tilde{A}$ , respectively.

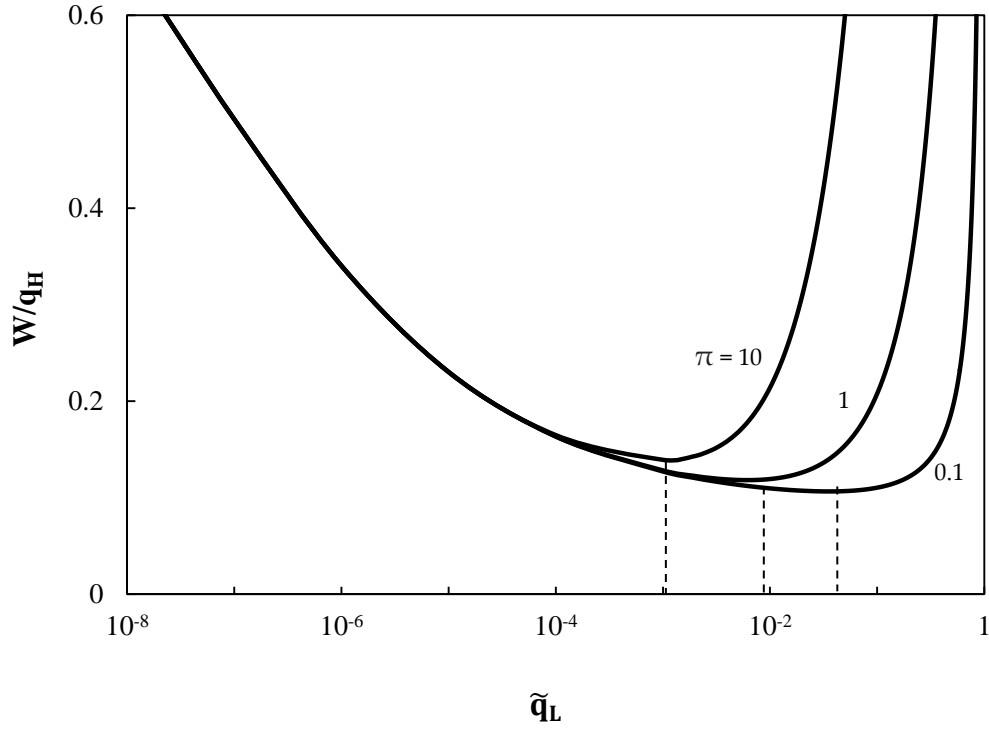
Alternatively, we can express the  $K$  constraint as

$$\tilde{K} = \tilde{q}_L + \pi \tilde{A} \quad (39)$$

where  $\tilde{K} = K/p_q$  and  $\pi = p_A/p_q$  are two known constants.

Combining Eqs. (26) and (39) we find that the total power requirement  $W/q_H$  has one degree of freedom,  $\tilde{q}_L$  or  $\tilde{A}$ . To illustrate this, we considered the numerical case  $\tilde{K} = 1$ ,  $\varepsilon = 0.1$ ,  $T_L/T_H = 0.95$ , and plotted  $W/q_H$  versus  $\tilde{q}_L$ . The result is shown in Fig. 11: a certain heat pump size corresponds to minimum total power requirement for a given price ratio  $\pi$ .

Figure 12 shows the variation of the minimum power requirement for the heat pump  $(W/q_H)_{\min}$  and its corresponding optimum heat pump size  $\tilde{q}_{L,opt}$  both as functions of the price ratio.



**Figure 11: The total power requirement  $W/q_H$  versus the heat pump size  $\tilde{q}_L$  when the total cost is constrained.**

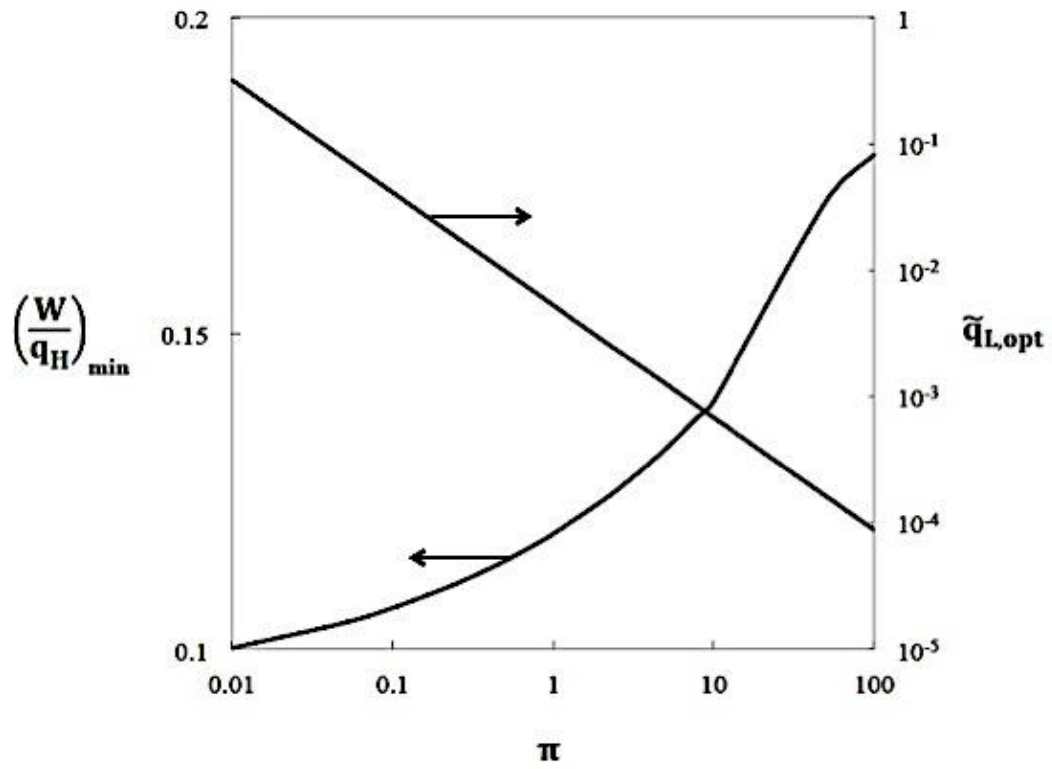


Figure 12: The variation of the minimum power requirement  $(W/q_H)_{\min}$  and its corresponding optimum heat pump size  $\tilde{q}_{L,opt}$  with the cost ratio  $\pi$ .

## 7. Conclusion

In this paper, we showed analytically the performance of the refrigeration and heat pump systems. We also stressed the fact that larger installations are more efficient. The performance of the ground-coupled heat pump is strongly affected by the size of the heat pump by it self and the size of the ground heat exchanger. The total power requirement for the heat pump decreases as either the size of the heat pump or the size of the ground heat exchanger increases. Central installations are more efficient than individual installations. The percentage of savings by switching from individual to central installations increase as the number of users increase. There is an optimum size for the heat pump that corresponds to minimum power consumption under a cost constraint. We also showed that, as the price of the heat pump become less than the price of the ground heat exchanger, the optimum heat pump size increases and the corresponding minimum power requirement decreases.

## References

- [1] A. Bejan, *Advanced Engineering Thermodynamics* (3rd ed.), Wiley, Hoboken, 2006, pp. 534-536.
- [2] T.R. Strohbridge, *Cryogenic refrigerators: an updated survey*, NBS Tech. Note (U.S.) No. 655, June 1974.
- [3] A.S. Lebedev, S.V. Kostennikov, Trends in increasing gas-turbine units efficiency, *Therm. Eng.* 55 (2008) 461–468.
- [4] S. Lorente, A. Bejan, B.S. Yilbas, A.Z. Sahin, “The Effect of Size on Efficiency: Power Plants and Vascular Designs”, *Int. J. Heat Mass Transfer* 54 (2011) 1475-1481.
- [5] K. Sarica, I. Or, Efficiency assessment of Turkish power plants using data envelopment analysis, *Energy* 32 (2007) 1484–1499.
- [6] Y.S. Kim, S. Lorente, A. Bejan, Distribution of size in steam turbine power plants, *Int. J. Energy Res.* 33 (2009) 989–998.
- [7] J. T. Chung, and J. M. Choi, Design and performance study of the ground-coupled heat pump system with an operating parameter, *Int. J. Renewable Energy* 42 (2012) 118–124.
- [8] S. Lorente, A. Bejan, K. Al-Hinai, A.Z. Sahin, B.S. Yilbas, Constructal design of distributed energy systems: solar power and water desalination, *Int. J. Heat and Mass Transfer* 55 (2012) 2213-2218.
- [9] E. Battisti, G. M. Casolino, F. Rossi, M. Russo, Economical considerations about combined cycle power plant control in deregulated markets. *Mario. Int. J. Elec. Power* 28 (2006) 284-292.
- [10] G.W. Wilson, T. Korakianitis, *The Design of High-efficiency Turbomachinery and Gas Turbines* (2nd ed.) Prentice Hall, Upper Saddle River, NJ (1998) p. 146.
- [11] Copeland Corporation, Motor horsepower vs. compressor efficiency, *Application Engineering Bulletin AE-1274*, January 1, 1985, Tables I and II.



- [12] Pacific Gas and Electric Company, Agricultural pumping efficiency improvements, Application Note, April 25, 1997, Table 2.
- [13] P. Fahlén, Capacity control of heat pumps, Federation of European Heating, Ventilation and Air Conditioning Associations Journal, October 2012.
- [14] J. Åström, Investigation of issues related to electrical efficiency improvements of pump and fan drives in buildings, Gothenburg, Federation of European Heating, Ventilation and Air Conditioning Associations Journal, October 2012.
- [15] C.-C. Chuang, D.-C. Sue, Performance effects of combined cycle power plant with variable condenser pressure and loading, *Energy* 42 (2006) 1793-1801.
- [16] M. Variny, O. Mierka, Improvement of part load efficiency of a combined cycle power plant provisioning ancillary services, *Applied Energy* 86 (2009) 888–894.
- [17] Y.-H. Song, Y. Akashi, J.-J. Yee, A study on the energy performance of a cooling plant system: Air-conditioning in a semiconductor factory, *Energy and Buildings* 40 (2008) 1521-1528.
- [18] V.R. Tarnawski, W.H. Leong, T. Momose, Y. Hamada, Analysis of ground source heat pumps with horizontal ground heat exchangers for northern Japan, *Int. J. Renewable Energy* 34 (2009) 127-134.
- [19] P. Vogt, *Dictionary of Statistics & Methodology: A Nontechnical Guide for the Social. Sciences*, Sage Publications, Thousand Oaks, CA, 2005.
- [20] T. T. Soong, *Fundamentals of Probability and Statistics for Engineers*, Wiley, Hoboken, 2004.